Revisiting Disjunctive Syllogism and *Ex falso*

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Abstract
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The relation between *ex falso* and *disjunctive syllogism*, or even the justification of *ex falso* based on disjunctive syllogism, is an old topic in the History of Logic (see [2], [3], [4]). This old topic reappears in contemporary Logic since the introduction of *Minimal logic* by Johansson(see [6], [10], [11]). The disjunctive syllogism seems to be part of our general non-problematic inferential practices and superficially it doesn't seem to be related to or to depend on our acceptance of the *ex falso* rule; on the other hand, the general validity of the *ex falso* has been subjected to dispute. We know that the acceptance of the *ex falso* is a sufficient condition for the acceptance of the disjunctive syllogism and that the acceptance of the *Disjunctive-syllogism rule* implies the acceptance of the *ex falso*, as the following simple derivations in an intuitionistic natural deduction system (see [1], [5]) extended with the *Disjunctive-syllogism rule* show:

$$\cfrac{(A \vee B) \qquad [A]^1 \qquad \cfrac{\cfrac{[B]^2 \qquad \neg B}{\cfrac{\bot}{A} \bot_i} \neg_{Elimination}}{}}{A} \vee_{Elimination} {}^{1,\ 2}$$

$$\cfrac{\cfrac{\cfrac{A}{(A \vee B)} \qquad \neg A}{B}}{} \text{Disjunctive-syllogism rule}$$

The interesting question is: is the ex falso really a necessary condition for the acceptance of the disjunctive syllogism? The aim of the present paper is to discuss some possible ways to define systems that combines the preservation of the disjunctive syllogism with the rejection of the *ex falso*. In the final part of the paper we discuss some interesting similarities and differences between our approach and Neil Tennant's relevantist approach ([6], [7], [8], [9]) to the same topic.

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